Some tips for doing trigonometric proofs

Trig is an extremely important niche of mathematics, particularly in geometrical and physical applications; basically all physics and engineering would be more or less impossible without a solid theory of trigonometry! One area of this topic that I struggled with particularly, and a lot of others I know did as well, was constructing trigonometric proofs. You know the ones I mean – those “Prove <some really messed up equation> = <like, sin(x) or 1 or something really simple>” questions. One of the reasons these questions are often challenging is that, unlike much of the content covered in the HSC course, constructing proofs is generally not algorithmic, and it isn’t always clear how to start.

So, how do we do these if there isn’t really any method? Well, there are techniques and tricks we can employ which may lead us in the correct direction. These are some of the ones I use when doing these questions.

HOW TO START

These questions appear in the form “Prove A = B”. There are generally two main techniques for attacking these types of problems:

1. Start with one side e.g. “LHS = A”, and show that “LHS = B”. I generally choose the side that looks more complicated, as it can generally be simplified more.
2. Start with “LHS = A” and show that “LHS = C”, for some expression C, and then write “RHS = B” and show that RHS = C as well.

These are both perfectly valid and equivalent methods. Emphatically do NOT start with the statement “A=B” and then “prove” that “1=1”, or “0=0” or something equivalent. This is incorrect (and it peeves markers [and me] like no end)!\*\* It is significantly better practice (and more technically correct) to manipulate either side of the equation individually.

THINGS THAT CAN HELP

Ok, so those are a couple of places to start, but we haven’t proved anything yet; all we’ve done is rewrite the question. Where can we go from here? Here are a couple of things that can often be helpful.

* COMBINE THINGS INTO SINGLE FRACTIONS: Common denominators are useful things to have and they can make expressions simpler.
* FACTORISE: Often factorising more complicated expressions can allow for cancellation of common factors in fractions, or simplify expressions. Often factorising can reveal something like a “sin2(x)+cos2(x)” that might not have been obvious before. Identities like perfect squares, difference of squares and, less commonly (but still important!) sum and difference of cubes are particularly handy here.
* TURN THINGS INTO SINES AND COSINES: By this point, you’ve been introduced to at least six different trig ratios: the three standard ones and three reciprocal relations. But you only need two in order to express all of these: sin and cos. This can be advantageous: a horrific expression of cosec and cot can be simplified through these three things into something in terms of sin and cos, which is often more manageable.
* REMEMBER YOUR IDENTITIES: Though it might seem a bit obvious, know your trig identities back to front. There are a couple that are especially important, as they are the most common:
  + sin2(x) + cos2(x) = 1 (the Pythagorean identity)
  + tan(x) = sin(x) / cos(x) (follows from definition)
  + cot(x) = cos(x) / sin(x) (similarly)
  + Reciprocal ratio definitions.

Generally, double angle formulae and complementary angle identities are less common and it is often more obvious when they need to be used.

SLIGHTLY LESS COMMON TRICKS

These are a couple of sneaky things that occasionally crop up the existence of which is sometimes handy to know.

* MULTIPLY BY ONE: Remember rationalising the denominator? We multiply the top and bottom of a fraction by the same thing. This is the same, but with trigonometric expressions.
* REWRITING ONE: Very occasionally, one can reverse the Pythagorean identity. Usually It is used to get rid of pesky sin2(x) + cos2(x) terms, but sometimes one can replace 1 with sin2(x) + cos2(x).

These are both very situational, but are sometimes useful to be aware of.

SUMMARY

The best way to master this, as with much of HSC maths, is to practice as wide a range of questions as possible. Build up your toolbox of techniques – these mine, and they served me well – the more techniques and tricks you are exposed to, the more tools you have at your disposal. And, if you’re in doubt about where to go: firstly, make sure your working up until that point is correct – often a single misstep can lead you into a world of pain, and persevering through an incorrect solution is unadvisable and time-inefficient. Secondly, try stuff (time permitting, of course)! Don’t be afraid to scribble in the margins. The more tools you have, the more things you can try.